Verified computations for hyperbolic 3-manifolds

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Recent progress in field of 3-manifold topology has confirmed that each 3-manifold can be decomposed in to pieces that admit a geometric structure modelled on the quotient of one of eight simply connected spaces (for further background see the references below). By most accounts, the most common, and yet least understood of these geometric structures is the hyperbolic structure. A manifold M admits a hyperbolic structure if M is homeomorphic to the quotient of $\mathbb{H}^3$ by some discrete subgroup of $PSL(2, \mathbb{C})$. Much of the power of Thurston's work (mentioned below) is that it allows one to compute topological invariants from geometric information.

This observation motives the following observation, the problem of determining invariants of the hyperbolic structure of a 3-manifold can be reduced have an exact description a set of shapes of tetrahedra subject to some constraints. However, as exact arithmetic is often expensive to compute verified computations for these shapes often prove to be more useful. After providing the necessary background (no prior knowledge of 3-manifold topology is assumed), this talk will focus on new numerical verification scheme which rigorously shows the existence of a hyperbolic structure and allows for the computation of further invariants.

References:

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