Orbital stability investigation for travelling waves in a nonlinearly supported beam

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We consider the fourth-order problem

$$\varphi_{tt} + \varphi_{xxxx} + f(\varphi) = 0, \quad (x,t) \in \mathbb{R} \times \mathbb{R}^+, \tag{1}$$

with a nonlinearity f vanishing at 0. Solitary waves $\varphi = u(x+ct)$ satisfy the ODE

$$u'''' + c^2 u'' + f(u) = 0 \quad \text{on } \mathbb{R}, \tag{2}$$

and for the case $f(u) = e^u - 1$, the existence of at least 36 travelling waves was proved in [1] by computer assisted means.

We investigate the orbital stability of these solutions via computation of their Morse indicies, which heavily relies on spectral bounds for the linearized operator, and using results from [2] and [3]. We make use of both analytical and computer-assisted techniques.

References:

[1] B. Breuer, J. Horák, P. J. McKenna, M. Plum, A computer-assisted existence and multiplicity proof for travelling waves in a nonlinearly supported beam, *Journal of Differential Equations*, 224, pp. 60-97, 2006.

[2] J. Shatah and W. Strauss, Stability Theory of Solitary Waves in the Presence of Symmetry, I, *Journal of Functional Analysis*, 74, pp. 160-197, 1987.

[3] J. Shatah and W. Strauss, Stability Theory of Solitary Waves in the Presence of Symmetry, II, *Journal of Functional Analysis*, 94, pp. 308-348, 1990.