A Computer-assisted multiplicity proof for a semilinear elliptic boundary value problem

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We consider positive solutions of $-\Delta u - u^p = 0$ in $\Omega \subset \mathbb{R}^N,\ u = 0$ on $\partial\Omega$, $1 < p < \frac{N+2}{N-2}$ ($p > 1$ in case $N = 2$), where Ω is a domain with a hole. It has been proven that multiple solutions to this problem exist in case of Ω being an annulus $\Omega_R = \{x \in \mathbb{R}^N : \ R < |x| < R + 1\}$ with $R > 0$ sufficiently large. It is moreover known that the number of (rotationally non-equivalent) positive solutions tends to infinity as $R \to \infty$. Similar results, which however are all of asymptotic nature, are known for other annulus-like domains.

We consider the problem in case of the domain $\Omega_t = (-1, 1)\backslash[-t, t] \subset \mathbb{R}^2$, where $t \in (0, 1)$, $p = 3$ and ask similar questions, with the additional demand of quantification: Is there a solution similar to the radial one in the annulus case? If yes, for which size of the hole does bifurcation from this solution occur, and how many solutions exist for certain sizes of the hole?

We try to answer these questions by computer-assistance: We compute approximate solutions and use a fixed point argument to enclose true solutions nearby. To verify the assumptions needed for the fixed point theorem we make heavy use of the computer.