(1) $f(x) = |x|$ $(-\pi \le x \le \pi)$ を $f(x+2\pi) = f(x)$ によって周期関数に拡張したものを図にしめせ

$f(x) = |x|$ $(-\pi \le x \le \pi)$

$f(x)$ は偶関数であるから $b_n = 0$ $(n = 1, 2, 3....)$

$a_0 = 2/\pi \int_0^\pi x\,dx = \pi$

$a_n = 2/\pi \int_0^\pi x\cos nx\,dx$

$= 2/n\pi \left([x\sin nx]^\pi{}_0 - \int_0^\pi \sin nx\,dx\right)$

$= 2\{(-1)^n - 1\}/\pi n^2$

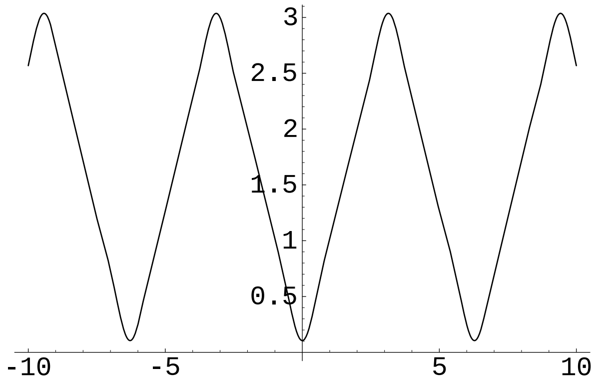
$= 0$ (nは偶数)

$= -4/\pi n^2$ (nは奇数)

よって $f(x)$ のフーリエ級数展開は次のようになる

$$f[x\_] := \frac{\pi}{2} - \frac{4}{\pi}\left(\sum_{k=1}^{3} \frac{1}{(2k-1)^2} \mathrm{Cos}[(2k-1)x]\right)$$

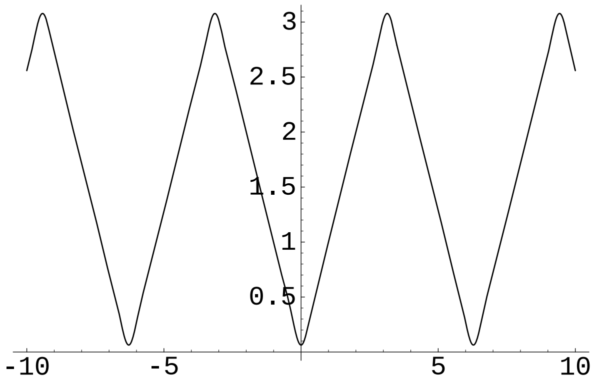
```
Plot[f[x], {x, -10, 10}]
```



- Graphics -

$$f[x\_] := \frac{\pi}{2} - \frac{4}{\pi}\left(\sum_{k=1}^{5} \frac{1}{(2k-1)^2} \mathrm{Cos}[(2k-1)x]\right)$$

```
Plot[f[x], {x, -10, 10}]
```



- Graphics -

(2) $g(x) = 1\ (0 \le x \le \pi)$,
$= 0\ (\pi \le x \le 2\pi)$ を $g(x+2\pi) = g(x)$ によって周期関数に拡張したものを図にしめせ

$g(x)$ は奇関数だから $a_n = 0\ (n = 1, 2, 3....)$

$$a_0 = \int_0^{2\pi} f(x)\, dx = 1$$

$$b_n = 1/\pi \left(\int_0^{\pi} 1 \sin nx\, dx + \int_{\pi}^{2\pi} 0 \sin nx\, dx\right)$$

$$= 1/\pi\ (1 - \cos n\pi / n)$$

$$= 1 - |-1|^n / n\pi$$

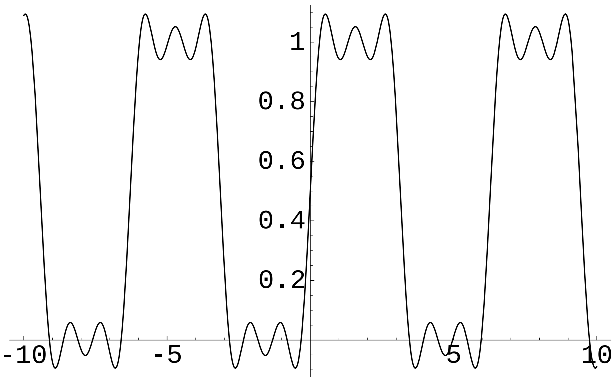
$= 2/n\pi$ (nは奇数)

$= 0$ (nは偶数)

よってg(x)のフーリエ級数展開は次のようになる

$$g[x\_] := \frac{1}{2} + \frac{2}{\pi}\left(\sum_{k=1}^{3} \frac{1}{2k-1} \mathrm{Sin}[(2k-1)\,x]\right)$$

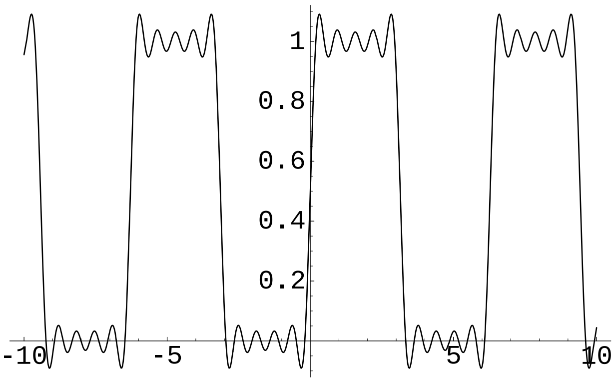
```
Plot[g[x], {x, -10, 10}]
```



- Graphics -

$$g[x\_] := \frac{1}{2} + \frac{2}{\pi}\left(\sum_{k=1}^{5}\frac{1}{2k-1}\,\mathrm{Sin}[(2k-1)\,x]\right)$$

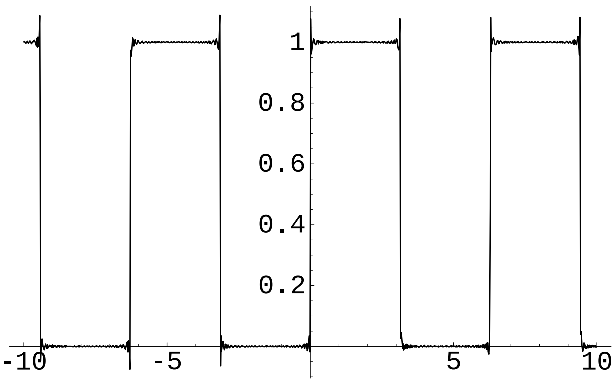
```
Plot[g[x], {x, -10, 10}]
```



- Graphics -

$$g[x\_] := \frac{1}{2} + \frac{2}{\pi}\left(\sum_{k=1}^{100}\frac{1}{2k-1}\,\mathrm{Sin}[(2k-1)\,x]\right)$$

```
Plot[g[x], {x, -10, 10}]
```



- Graphics -