

問題：

- (1)  $f(x) = |x|$  ( $-\pi \leq x < \pi$ ) を  
 $f(x+2\pi) = f(x)$  によって周期関数に拡張したもの。  
 (2)  $f(x) = 1$  ( $0 \leq x < \pi$ ),  $f(x) = 0$  ( $\pi \leq x < 2\pi$ ) を  
 $f(x+2\pi) = f(x)$  によって周期関数に拡張したもの。

解答：

(1)

$$\begin{aligned} a_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} |x| * \cos[nx] \, dx \\ &= \frac{2}{\pi} \int_0^{\pi} x * \cos[nx] \, dx \\ &= \frac{2}{\pi n^2} (\cos[n\pi] - 1) \end{aligned}$$

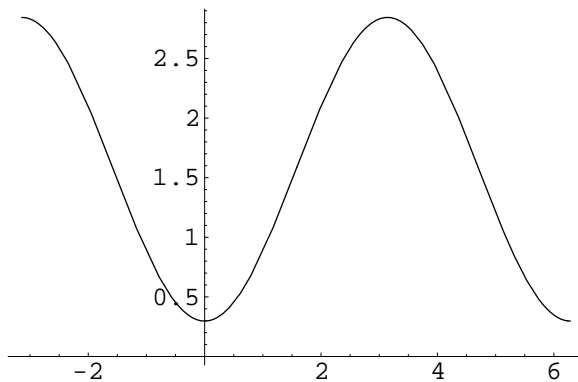
$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} |x| * \sin[nx] \, dx$$

今、 $a_n$  は偶関数・ $b_n$  は奇関数であるから、フーリエ級数展開は

$$f(x) = \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{2}{\pi n^2} (\cos[n\pi] - 1) * \cos[nx] \quad \text{となる。}$$

$n = 1$  のときのグラフ

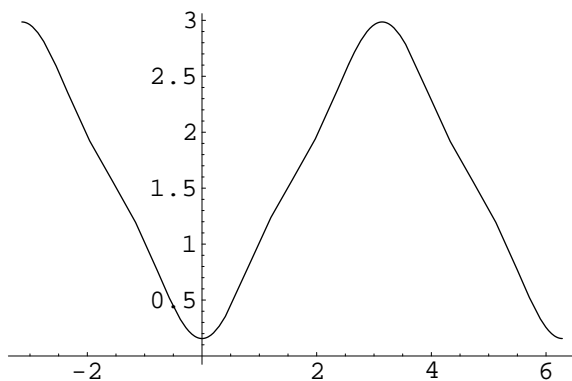
$$\text{Plot}\left[\frac{\pi}{2} + \left(-\frac{4}{\pi}\right) * \cos[x], \{x, -\pi, 2\pi\}\right]$$



- Graphics -

$n = 3$  のときのグラフ

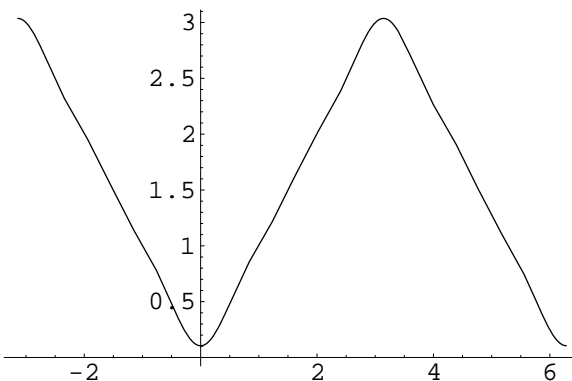
$\text{Plot}\left[\frac{\pi}{2} + \left(-\frac{4}{\pi}\right) * \text{Cos}[x] + \left(-\frac{4}{9\pi}\right) * \text{Cos}[3x], \{x, -\pi, 2\pi\}\right]$



- Graphics -

**n = 5 のときのグラフ**

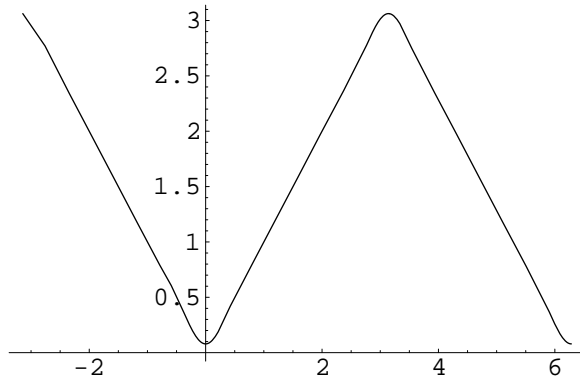
$\text{Plot}\left[\frac{\pi}{2} + \left(-\frac{4}{\pi}\right) * \text{Cos}[x] + \left(-\frac{4}{9\pi}\right) * \text{Cos}[3x] + \left(-\frac{4}{25\pi}\right) * \text{Cos}[5x], \{x, -\pi, 2\pi\}\right]$



- Graphics -

**n = 7 のときのグラフ**

Plot[ $\frac{\pi}{2} + \left(-\frac{4}{\pi}\right) * \text{Cos}[x] +$   
 $\left(-\frac{4}{9\pi}\right) * \text{Cos}[3x] + \left(-\frac{4}{25\pi}\right) * \text{Cos}[5x] + \left(-\frac{4}{49\pi}\right) * \text{Cos}[7x], \{x, -\pi, 2\pi\}$ ]



- Graphics -

( 2 )

$$\begin{aligned} a_n &= \frac{1}{\pi} \int_0^\pi \text{Cos}[nx] \, dx \dots (B) \\ &= \frac{1}{\pi n} * \text{Sin}[\pi n] \\ &= 0 \end{aligned}$$

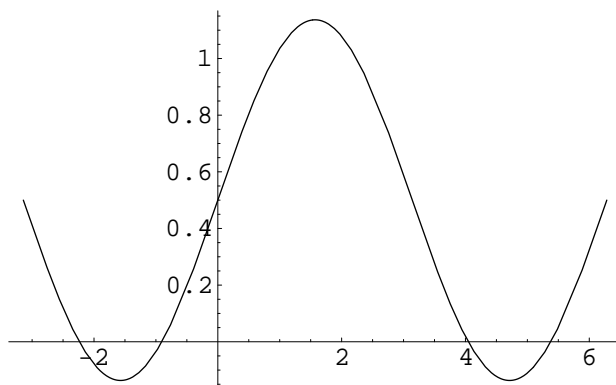
$$\begin{aligned} b_n &= \frac{1}{\pi} \int_0^\pi \text{Sin}[nx] \, dx \\ &= \frac{1}{\pi n} (1 - \text{Cos}[\pi n]) \end{aligned}$$

したがって、フーリエ級数展開は、

$$f(x) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{1}{\pi n} (1 - \text{Cos}[\pi n]) * \text{Sin}[nx] \quad \text{となる。}$$

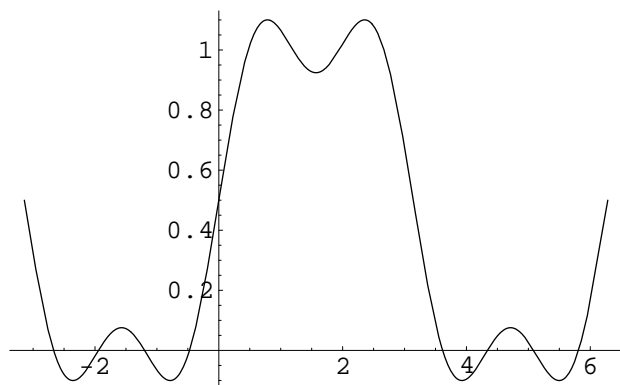
n = 1 のときのグラフ

Plot[ $\frac{1}{2} + \frac{2}{\pi} * \text{Sin}[x], \{x, -\pi, 2\pi\}$ ]



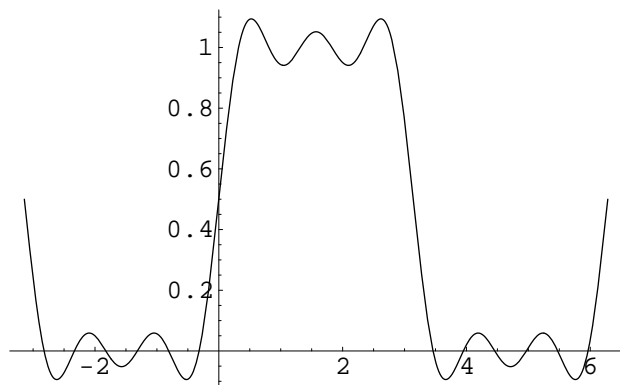
- Graphics -

n = 3 のときのグラフ

$$\text{Plot}\left[\frac{1}{2} + \frac{2}{\pi} * \text{Sin}[x] + \frac{2}{3\pi} * \text{Sin}[3x], \{x, -\pi, 2\pi\}\right]$$


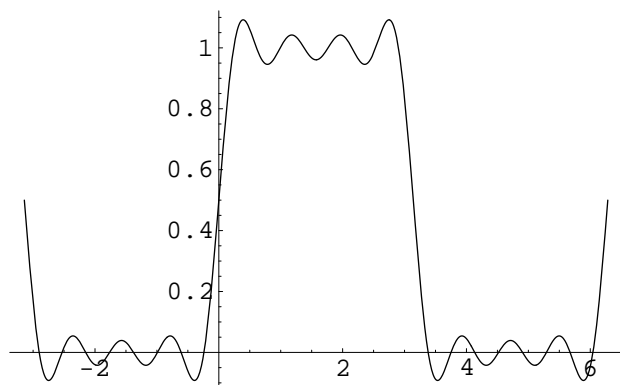
- Graphics -

**n = 5 のときのグラフ**

$$\text{Plot}\left[\frac{1}{2} + \frac{2}{\pi} * \text{Sin}[x] + \frac{2}{3\pi} * \text{Sin}[3x] + \frac{2}{5\pi} * \text{Sin}[5x], \{x, -\pi, 2\pi\}\right]$$


- Graphics -

**n = 7 のときのグラフ**

$$\text{Plot}\left[\frac{1}{2} + \frac{2}{\pi} * \text{Sin}[x] + \frac{2}{3\pi} * \text{Sin}[3x] + \frac{2}{5\pi} * \text{Sin}[5x] + \frac{2}{7\pi} * \text{Sin}[7x], \{x, -\pi, 2\pi\}\right]$$


- Graphics -